

# SCHWARZSCHILD SOLUTION ON A SPACE-TIME WITH TORSION

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**ABSTRACT.** We obtain the Schwarzschild solution based on teleparallel gravity (TG) theory formulated in a space-time with torsion only. The starting point is the Poincaré gauge theory (PGT). The general structure of TG and its connection with general relativity (GR) are presented and the Schwarzschild solution is obtained by solving the field equations of TG. Most of calculations are performed using the GRTensorII package, running on the MapleV platform.

## 1. INTRODUCTION

In general relativity (GR), the curvature of the space-time is used to describe the gravitation. The geometry of the space-time replaces the concept of force. On the other hand, teleparallel gravity (TG) attributes gravitation to torsion [1,2] and it is a gauge theory for the group of space-time translations. The gravitational interactions are described in TG by forces, similar to the Lorentz forces in electrodynamics. Therefore, the gravitational interactions can be described alternatively in terms of curvature as is usually done in GR, or in terms of torsion as in TG. It is believed that requesting a curved or a torsioned space-time to describe gravity is a matter of convention [3].

In this paper we obtain the Schwarzschild solution in the case of TG. The general structure of TG and its connection with GR are presented and the Schwarzschild solution is obtained by solving the field equations. Section 2 contains a review of Poincaré gauge theory (PGT) on a Minkowski space-time as base manifold. In Section 3 a non-symmetric connection is constructed starting with gauge fields of PGT. The field equations of TG are then obtained in the Section 4 under a general form. The Schwarzschild solution of these equations and the analytical program are presented in Section 5. A comparison with GR case is also presented. The conclusions and some remarks are presented in Section 6.

## 2. POINCARÉ GAUGE THEORY

We will denote the generators of Poincaré group  $P$  by  $\{P_a, M_{ab}\}$ , where  $a, b = 0, 1, 2, 3$ . Here,  $P_a$  are the generators of space-time translations and  $M_{ab} = -M_{ba}$  are the generators of the Lorentz rotations. Suppose now that  $P$  is a gauge group for gravitation [4]. Correspondingly, we introduce the 1-form potential  $A$  with values in Lie algebra of the Poincaré group, defined by formula:

$$(1) \quad A = e^a P_a + \frac{1}{2} \omega^{ab} M_{ab}$$

where  $e^a = e^a_\mu dx^\mu$  and  $\omega^{ab} = \omega^{ab}_\mu dx^\mu$  are ordinary 1-forms. The 1-form defines a connection on the space-time  $M_4$  of our gauge model with  $e^a_\mu$  and  $\omega^{ab}_\mu$  as gauge

fields. The 2-form of curvature  $F$  is given by the expression

$$(2) \quad F = dA + \frac{1}{2} [A, A]$$

Inserting (1) in (2) and identifying the result with the definition

$$(3) \quad F = T^a P_a + \frac{1}{2} R^{ab} M_{ab}$$

we obtain the 2-forms of torsion  $T^a$  and of curvature  $R^{ab}$  in the form

$$(4) \quad T^a = de^a + \omega^a{}_b \wedge e^b,$$

and respectively

$$(5) \quad R^a{}_b = d\omega^a{}_b + \omega^a{}_c \wedge \omega^c{}_b.$$

We use the Minkowski metric  $\eta_{ab} = \text{diag}(1, -1, -1, -1)$  on the Poincaré group manifold to raise and lower the latin indices  $a, b, c$ . Written on the components, the equations (4) and (5) give:

$$(6) \quad T^a_{\mu\nu} = \partial_\mu e_\nu^a - \partial_\nu e_\mu^a + (\omega_\mu^{ab} e_\nu^c - \omega_\nu^{ab} e_\mu^c) \eta_{bc},$$

and respectively

$$(7) \quad R^{ab}{}_{\mu\nu} = \partial_\mu \omega_\nu^{ab} - \partial_\nu \omega_\mu^{ab} + (\omega_\mu^{ac} \omega_\nu^{db} - \omega_\nu^{ac} \omega_\mu^{db}) \eta_{cd}.$$

The quantity  $T^a_{\mu\nu}$  is the torsion tensor and the quantity  $R^{ab}{}_{\mu\nu}$  is the curvature tensor of the connection  $A$  defined by the equation (1). The connection  $A$  defines a structure Einstein-Cartan (EC) on the space-time and we will denote the corresponding space by  $U_4$ . This space have both torsion and curvature. In the next Section, we will consider the case of a space-time with non-null torsion and vanishing curvature to construct the teleparallel theory of gravity ( $TG$ ).

### 3. TELEPARALLEL GRAVITY

We interpret  $e_\nu^a$  as tetrad fields and  $\omega_\nu^{ab}$  as spin connection. A model of gauge theory based on Poincaré group and implying only torsion can be obtained choosing  $\omega_\nu^{ab} = 0$ . Then the curvature tensor  $R^{ab}{}_{\mu\nu}$  vanishes and the torsion in equation (6) becomes:

$$(8) \quad T^a{}_{\mu\nu} = \partial_\mu e_\nu^a - \partial_\nu e_\mu^a$$

Expressed in a coordinate basis, this tensor has the components:

$$(9) \quad T^\rho{}_{\mu\nu} = e_a{}^\rho \partial_\mu e_\nu^a - e_a{}^\rho \partial_\nu e_\mu^a,$$

where  $e_a{}^\rho$  is the inverse of  $e^a{}_\rho$ .

Now, we define the Cartan connection  $\Gamma$  on the space-time  $M_4$  with nonsymmetric coefficients

$$(10) \quad \Gamma^\rho{}_{\mu\nu} = e_a{}^\rho \partial_\nu e_\mu^a.$$

This definition is suggested by the expression (9) of the torsion components. Therefore, the connection  $\Gamma$  has the torsion given by the usually formula

$$(11) \quad T^\rho{}_{\mu\nu} = \Gamma^\rho{}_{\nu\mu} - \Gamma^\rho{}_{\mu\nu}.$$

With respect to the connection  $\Gamma$ , the tetrad field is parallel, that is:

$$(12) \quad \nabla_\mu e^a{}_\nu = \partial_\mu e_\nu^a - \Gamma^\rho{}_{\nu\mu} e^a{}_\rho = 0.$$

Curvature and torsion have to be considered as properties of the connections and therefore many different connections are allowed on the same space-time [5]. For example, starting with the tetrad field  $e^a{}_\mu$  we can define the Riemannian metric:

$$(13) \quad g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^a{}_\nu.$$

Then, we can introduce the Levi-Civita connection

$$(14) \quad \overset{\circ}{\Gamma}{}^\sigma_{\mu\nu} = \frac{1}{2} g^{\sigma\rho} (\partial_\mu g_{\rho\nu} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}).$$

This connection is metric preserving:

$$(15) \quad \overset{\circ}{\nabla}_\rho g^{\mu\nu} = \partial_\rho g^{\mu\nu} + \overset{\circ}{\Gamma}{}^\mu_{\sigma\rho} g^{\sigma\nu} + \overset{\circ}{\Gamma}{}^\nu_{\sigma\rho} g^{\sigma\mu} = 0.$$

The relation between the two connections  $\Gamma$  and  $\overset{\circ}{\Gamma}$  is

$$(16) \quad \Gamma^\sigma{}_{\mu\nu} = \overset{\circ}{\Gamma}{}^\sigma_{\mu\nu} + K^\sigma{}_{\mu\nu},$$

where

$$(17) \quad K^\sigma{}_{\mu\nu} = \frac{1}{2} (T_\mu{}^\sigma{}_\nu + T_\nu{}^\sigma{}_\mu - T^\sigma{}_{\mu\nu})$$

is the contortion tensor.

The curvature tensor of the Levi-Civita Connection  $\overset{\circ}{\Gamma}$  is:

$$(18) \quad \overset{\circ}{R}{}^\sigma{}_{\rho\mu\nu} = \partial_\mu \overset{\circ}{\Gamma}{}^\sigma_{\rho\nu} - \partial_\nu \overset{\circ}{\Gamma}{}^\sigma_{\rho\mu} + \overset{\circ}{\Gamma}{}^\sigma_{\tau\mu} \overset{\circ}{\Gamma}{}^\tau_{\rho\nu} - (\mu \leftrightarrow \nu)$$

Because the connection coefficients  $\overset{\circ}{\Gamma}{}^\sigma_{\mu\nu}$  are symmetric in the indices  $\mu$  and  $\nu$ , its torsion is vanishing. Therefore, the Levi-Civita connection  $\overset{\circ}{\Gamma}$  have non-null curvature, but no torsion. Contrarily, the Cartan connection  $\Gamma$  presents torsion, but no curvature. Indeed, using the definition (10), we can verify that the curvature of the connection  $\Gamma$  vanishes identically:

$$(19) \quad R^\sigma{}_{\rho\mu\nu} = \partial_\mu \Gamma^\sigma_{\rho\nu} - \partial_\nu \Gamma^\sigma_{\rho\mu} + \Gamma^\sigma_{\tau\mu} \Gamma^\tau_{\rho\nu} - (\mu \leftrightarrow \nu) \equiv 0.$$

Then, substituting (16) into the expression (19), we obtain:

$$(20) \quad R^\sigma{}_{\rho\mu\nu} = \overset{\circ}{R}{}^\sigma{}_{\rho\mu\nu} + Q^\sigma{}_{\rho\mu\nu} \equiv 0,$$

where

$$(21) \quad Q^\sigma{}_{\rho\mu\nu} = D_\mu K^\sigma{}_{\rho\nu} + \Gamma^\sigma{}_{\tau\nu} K^\tau{}_{\rho\mu} - (\mu \leftrightarrow \nu)$$

is the non-metricity tensor. Here

$$(22) \quad D_\mu K^\sigma{}_{\rho\nu} = \partial_\mu K^\sigma{}_{\rho\nu} + \Gamma^\sigma{}_{\tau\mu} K^\tau{}_{\rho\nu} - \Gamma^\sigma{}_{\tau\nu} K^\tau{}_{\rho\mu}$$

is the teleparallel covariant derivative.

The equation (20) has an interesting interpretation [6]: the contribution  $\overset{\circ}{R}{}^\sigma{}_{\rho\mu\nu}$  coming from the Levi-Civita connection  $\overset{\circ}{\Gamma}$  compensates exactly the contribution  $Q^\sigma{}_{\rho\mu\nu}$  coming from the Cartan connection  $\Gamma$ , yielding an identically zero Cartan curvature tensor  $R^\sigma{}_{\rho\mu\nu}$ .

Now, according to GR theory, the dynamics of the gravitational field is determined by the Lagrangian [6]:

$$(23) \quad L_{GR} = \frac{\sqrt{-g} c^4}{16\pi G} \overset{\circ}{R},$$

where  $\overset{\circ}{R} = g^{\mu\nu} \overset{\circ}{R}_{\mu\rho\nu}$  is the scalar curvature of the Levi-Civita connection  $\overset{\circ}{\Gamma}$ ,  $G$  is the gravitational constant and  $g = \det(g_{\mu\nu})$ . Then, substituting  $\overset{\circ}{R}$  as obtained from (20), one obtains up to divergences [6]:

$$(24) \quad L_{TG} = \frac{ec^4}{16\pi G} S^{\rho\mu\nu} T_{\rho\mu\nu} ,$$

where  $e = \det(e^a{}_\mu) = \sqrt{-g}$ , and

$$(25) \quad S^{\rho\mu\nu} = -S^{\rho\nu\mu} = \frac{1}{2} (K^{\mu\nu\rho} - g^{\rho\nu} T^{\sigma\mu}{}_\sigma + g^{\rho\mu} T^{\sigma\nu}{}_\sigma)$$

is a tensor written in terms of the Cartan connection only. The equation (24) gives the Lagrangian of the TG as a gauge theory of gravitation for the translation group.

It is proven [7] that the translational gauge theory of gravitation TG with the Lagrangian  $L_{TG}$  quadratic in torsion is completely equivalent to general relativity GR with usual Lagrangian  $L_{GR}$  linear in the scalar curvature. Therefore, the gravitation presents two equivalent descriptions: one GR in terms of a metric geometry and another one TG in which the underlying geometry is provided by a teleparallel structure.

In the next Section we will obtain the field equations of gravitation within TG theory.

#### 4. FIELD EQUATIONS

Taking the variation of the Lagrangian  $L_{TG}$  in Eq. (24) with respect to the gauge field  $e^a{}_\mu$ , one obtains the teleparallel version of the gravitational field equations:

$$(26) \quad \partial_\nu (e S_a{}^{\nu\rho}) - \frac{4\pi G}{c^4} (e j_a{}^\rho) = 0,$$

where  $S_a{}^{\nu\rho} = e_a{}^\mu S_\mu{}^{\nu\rho}$  and

$$S_\mu{}^{\nu\rho} = g_{\mu\tau} S^{\tau\nu\rho} = \frac{1}{4} (T_\mu{}^{\nu\rho} + T^\nu{}_\mu{}^\rho - T^\rho{}_\mu{}^\nu) - \frac{1}{2} (\delta_\mu{}^\rho T_\sigma{}^{\nu\sigma} - \delta_\mu{}^\nu T_\sigma{}^{\rho\sigma}).$$

The quantity  $j_a{}^\rho$  in Eq.(26) is the gauge gravitational current, defined analogous to the Yang-Mills theory:

$$(27) \quad j_a{}^\rho = \frac{1}{e} \frac{\partial L_{TG}}{\partial e_a{}^\rho} = -\frac{c^4}{4\pi G} e_a{}^\sigma S_\mu{}^{\nu\rho} T^\mu{}_{\nu\sigma} + \frac{1}{e} e_a{}^\rho L_{TG}.$$

The current  $j_a{}^\rho$  represents the energy-momentum of the gravitational field. The term  $e S_a{}^{\sigma\rho}$  is called superpotential in the sense that its derivative yields the gauge current  $e j_a{}^\rho$ . Due to the anti-symmetry of  $S_a{}^{\sigma\rho}$  in the indices  $\sigma$  and  $\rho$  the quantity  $e j_a{}^\rho$  is conserved as a consequence of the field equations, i.e.

$$(28) \quad \partial_\rho (e j_a{}^\rho) = 0.$$

Making use of Eq. (10) to express  $\partial_\rho e_a{}^\sigma$ , the field equations (26) can be written in a purely space-time form:

$$(29) \quad \frac{1}{e} \partial_\sigma (e S_\mu{}^{\sigma\rho}) - \frac{4\pi G}{c^4} (t_\mu{}^\rho) = 0,$$

where  $t_\mu{}^\rho$  is the canonical energy-momentum pseudo-tensor of the gravitational field [5], defined by the expression:

$$(30) \quad t_\sigma{}^\rho = \frac{c^4}{4\pi G} \Gamma^\mu{}_{\nu\sigma} S_\mu{}^{\nu\rho} + \frac{1}{e} \delta_\sigma{}^\rho L_{TG}.$$

It is important to notice that the canonical energy-momentum pseudo-tensor  $t_\mu{}^\rho$  is not simply the gauge current  $j_a{}^\rho$  with the Lorentz index "a" changed to the space-time index " $\mu$ ". It incorporates also an extra term coming from the derivative term of Eq. (26)

$$(31) \quad t_\sigma{}^\rho = e^a{}_\sigma j_a{}^\rho + \frac{c^4}{4\pi G} \Gamma^\mu{}_{\sigma\nu} S_\mu{}^{\nu\rho}.$$

Like the gauge current  $e j_a{}^\rho$ , the pseudo-tensor  $e t_\mu{}^\rho$  is conserved as a consequence of the field equation:

$$(32) \quad \partial_\rho (e t_\mu{}^\rho) = 0.$$

But, due to the pseudo-tensor character of  $t_\mu{}^\rho$ , this conservation law can not be expressed with a covariant derivative, in contrast with  $j_a{}^\rho$  case.

Using the previous results, we will prove in the next Section that the Schwarzschild solution can be obtained from the field equations (29) of the teleparallel theory of gravity.

## 5. SCHWARZSCHILD SOLUTION AND ANALYTICAL PROGRAM

Because we are looking for a spherically symmetric solution of the field equations, we will choose the Minkowski metric

$$(33) \quad ds^2 = dt^2 - dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

on the space-time manifold. The coordinates  $x^0, x^1, x^2, x^3$  correspond to  $ct, r, \theta, \varphi$  respectively. Then we will consider the gauge theory based on Poincaré group with  $\omega_\nu^{ab} = 0$  described in Section 3. The tetrad field  $e^a{}_\mu$  will be chosen under the form:

$$(34) \quad (e^a{}_\mu) = \begin{pmatrix} e^{A/2} & 0 & 0 & 0 \\ 0 & e^{B/2} & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & r \sin \theta \end{pmatrix},$$

where  $A = A(r)$  and  $B = B(r)$  are functions only of the 3D radius  $r$ . The inverse of  $e^a{}_\mu$  is therefore:

$$(35) \quad (e_a{}^\mu) = \begin{pmatrix} e^{-A/2} & 0 & 0 & 0 \\ 0 & e^{-B/2} & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & 0 & 0 & \frac{1}{r \sin \theta} \end{pmatrix}$$

The metric  $g_{\mu\nu} = \eta_{ab} e^a{}_\mu e^b{}_\nu$  will have then the form:

$$(36) \quad (g_{\mu\nu}) = \begin{pmatrix} e^A & 0 & 0 & 0 \\ 0 & -e^B & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2 \sin^2 \theta \end{pmatrix}$$

where the Eq. (33) have been used. The inverse of  $g_{\mu\nu}$  is evidently:

$$(37) \quad (g^{\mu\nu}) = \begin{pmatrix} e^{-A} & 0 & 0 & 0 \\ 0 & -e^{-B} & 0 & 0 \\ 0 & 0 & -\frac{1}{r^2} & 0 \\ 0 & 0 & 0 & -\frac{1}{r^2 \sin^2 \theta} \end{pmatrix}.$$

We use the above expressions to compute the coefficients  $\Gamma^\rho{}_{\mu\nu}$  of the Cartan connection, the components  $T_\mu{}^{\nu\rho}$  of the torsion tensor, of the tensor  $S_\mu{}^{\nu\rho}$  and of the canonical energy-momentum pseudo-tensor  $t_\mu{}^\rho$ . From this point at end we performed all the calculations using an analytical program conceived by us and which is given in Section 6. For example, the non-null components of the tensor  $T_\mu{}^{\nu\rho}$  are:

$$(38) \quad T_0{}^{01} = \frac{A'e^{-B}}{2}, \quad T_2{}^{21} = T_3{}^{31} = \frac{e^{-B}}{r}, \quad T_3{}^{32} = \frac{\cot\theta}{r^2},$$

where  $A' = \frac{dA}{dr}$  denote de derivative of the function  $A(r)$  with respect to the variable  $r$ . We will use the same notation for the derivative of  $B(r)$ , that is  $B' = \frac{dB}{dr}$ .

We list also the non-null components of the tensor  $S_\mu{}^{\sigma\rho}$  and of the canonical energy-momentum pseudo-tensor  $t_\mu{}^\rho$  of the gravitational field. Thus, for  $S_\mu{}^{\sigma\rho}$  we have:

$$S_0{}^{10} = \frac{e^{-B}}{r}, \quad S_0{}^{20} = S_1{}^{21} = \frac{\cot\theta}{2r^2}, \quad S_2{}^{12} = S_3{}^{13} = \frac{e^{-B}(rA' + 2)}{4r},$$

and for  $t_\mu{}^\rho$ :

$$\begin{aligned} t_0{}^0 &= t_1{}^1 = t_2{}^2 = t_3{}^3 = \frac{c^4}{4\pi G} \frac{e^{-B}(rA' + 1)}{2r^2}, \\ t_1{}^2 &= -\frac{c^4}{4\pi G} \frac{(A' + B')\cot\theta}{4r^2}, \quad t_2{}^1 = -\frac{c^4}{4\pi G} \frac{e^{-B}(rA' + 2)\cot\theta}{4r}. \end{aligned}$$

Now, using these components we obtain from (29) the following equations of gravitational field in  $TG$  theory:

$$(39) \quad e^{-B} \left( \frac{B'}{r} - \frac{1}{r^2} \right) + \frac{1}{r^2} = 0,$$

$$(40) \quad e^{-B} \left( \frac{A'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} = 0,$$

$$(41) \quad 2A'' + \left( \frac{2}{r} + A' \right) (A' - B') = 0,$$

where  $A'' = \frac{d^2A}{dr^2}$  is the second derivative of  $A(r)$  with respect to  $r$ . It is easy to verify that the third field equation (41) is a combination of the first two (39) and (40). Therefore, the equation (39) and (40) are the only independent field equations and they determine the two unknown functions  $A(r)$  and  $B(r)$ .

The solution of the equations (39) and (40) are [8]:

$$(42) \quad e^{-B} = e^A = 1 + \frac{\alpha}{r},$$

where  $\alpha$  is a constant of integration. It is known that the constant  $\alpha$  can be expressed by the mass  $m$  of the body which is the source of the gravitational field with spherically symmetry [8]:  $\alpha = -\frac{2Gm}{c^2}$ . Therefore, we obtain the Schwarzschild solution

$$(43) \quad ds^2 = \left( c^2 - \frac{2Gm}{r} \right) dt^2 - \frac{dr^2}{\left( 1 - \frac{2Gm}{c^2 r} \right)} - r^2 (d\theta^2 + \sin^2\theta d\varphi^2),$$

within the frame-work of  $TG$  theory of the gravitational field.

Finally, we emphasize again on the conclusion given in Ref. [2]: the gravitation presents two equivalent descriptions, one in terms of a metric geometry, and another one in which the underlying geometry is that provided by a teleparallel structure.

```

Program "TELEPARALLEL GRAVITY.MWS"
restart: grtw( );
grload (minkowski, 'spheric.mpl');
grdef('ev{^a miu}'); grcalc(ev(up,dn));
grdef('evinv{ a ^miu}'); grcalc(evinv(dn,up));
grdef('eta{a b}'); grcalc(eta(dn,dn));
grdef('etainv{^a ^b}'); grcalc(etainv(up,up));
grdef('ge{miu niu}:=ev{^a miu}*ev{^b niu}*eta{a b}'); grcalc(ge(dn,dn));
grdef('geinv{^miu ^niu}:=etainv{^a ^b}*evinv{a ^miu}*evinv{b ^niu}'); grcalc(geinv(up,up));
grdef('gama{^sigma miu niu}:=evinv{a ^sigma}*ev{^a miu,niu}');
grcalc(gama(up,dn,dn));
grdef('TORS{^sigma miu niu}:=gama{^sigma niu miu}-gama{^sigma miu niu}');
grcalc(TORS(up,dn,dn));
grdef('TS1{miu ^niu ^rho}:=ge{miu sigma}*geinv{^niu ^lambda}*geinv{^rho
^tau}*
TORS{^sigma lambda tau}'); grcalc(TS1(dn,up,up));
grdef('TS2{^niu miu ^rho}:=geinv{^rho ^sigma}*TORS{^niu miu sigma}'); gr-
calc(TS2(up,dn,up));
grdef('S{miu ^niu ^rho}:=(1/4)*(TS1{miu ^niu ^rho}+TS2{^niu miu ^rho}-
TS2{^rho miu
^niu}-(1/2)*(kdelta{miu ^rho}*TS1{sigma ^niu ^sigma}-kdelta{miu ^niu}*TS1{sigma
^rho ^sigma}'))');
grcalc(S(dn,up,up));
grdef('ed:=r^2*sin(theta)*exp((A(r)+B(r))/2); grcalc(e);
grdef('eS{lambda ^sigma ^rho}:=ed*S{lambda ^sigma ^rho}');
grcalc(eS(dn,up,up));
grdef('t{lambda ^rho}:=(c^4/(4*pi*G))*(gama{^miu niu lambda}*S{miu ^niu
^rho}+(1/4)*kdelta{lambda
^rho}*S{tau ^miu ^niu}*TORS{^tau miu niu}'))'); grcalc(t(dn,up));
grdef('EQ{lambda ^rho}:=(1/ed)*eS{lambda ^sigma ^rho,sigma}-(4*pi*G/c^4)*t{lambda
^rho}');
grcalc(EQ(dn,up)); grdisplay(-);

```

## 6. CONCLUDING REMARKS

We obtained the Schwarzschild solution within the teleparallel theory (TG) of gravity which is formulated in a space-time with torsion only. This can be interpreted as an indication that source of the torsion can be also the mass of the bodies that create the gravitational field, not only the spin. Therefore the torsion and curvature of the space-time is determined by the matter distribution in the considered region.

Most of the calculations have been performed using an analytical program conceived by us. The program allows to calculate the components of all quantities appearing in the model and to obtain also the equations of the gravitational field.

The TG theory can be used also to unify the gravitational field with other fundamental interactions (electromagnetic, weak and strong). Some results about this problem are given in our paper [9].

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